問題 4.1

1.　つぎの不定積分を求めよ．(以下，積分定数を省略する．)

(1)
$$\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x}+1}dx = \int \frac{1}{u^2+1}\frac{du}{dx}dx \qquad \left(u = e^x,\ \frac{du}{dx} = e^x\right)$$
$$= \int \frac{1}{u^2+1}dx = \mathrm{Tan}^{-1}u = \mathrm{Tan}^{-1}e^x$$

(4)
$$\int x\sqrt{1-x^2}dx = \int -\frac{1}{2}\sqrt{1-x^2}(-2x)dx = \int -\frac{1}{2}\sqrt{u}\ \frac{du}{dx}dx \qquad \left(u = 1-x^2,\ \frac{du}{dx} = -2x\right)$$
$$= \int -\frac{1}{2}\sqrt{u}\ du = -\frac{1}{3}u^{\frac{3}{2}} = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}$$

(5)
$$\int \frac{x}{(1+x^2)^3}dx = \int \frac{1}{2}\cdot\frac{2x}{(1+x^2)^3}dx = \int \frac{1}{2}\cdot\frac{1}{u^3}\frac{du}{dx}dx \qquad \left(u = 1+x^2,\ \frac{du}{dx} = 2x\right)$$
$$= \int \frac{1}{2}\cdot\frac{1}{u^3}du = -\frac{1}{4u^2} = -\frac{1}{4(1+x^2)^2}$$

(8)
$$\int \mathrm{Sin}^{-1}x dx = x\mathrm{Sin}^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx = x\mathrm{Sin}^{-1}x + \int \frac{1}{2}\cdot\frac{-2x}{\sqrt{1-x^2}}dx$$
$$= x\mathrm{Sin}^{-1}x + \int \frac{1}{2}\cdot\frac{1}{\sqrt{u}}\ \frac{du}{dx}dx \qquad \left(u = 1-x^2,\ \frac{du}{dx} = -2x\right)$$
$$= x\mathrm{Sin}^{-1}x + \int \frac{1}{2}\cdot\frac{1}{\sqrt{u}}\ du$$
$$= x\mathrm{Sin}^{-1}x + \sqrt{u} = x\mathrm{Sin}^{-1}x + \sqrt{1-x^2}$$

問題 4.1

2.　つぎの定積分の値を求めよ．

(4)
$$\int_0^{\sqrt{3}} \frac{1+x}{1+x^2}dx = \int_0^{\sqrt{3}} \frac{1}{1+x^2}dx + \int_0^{\sqrt{3}} \frac{1}{2}\cdot\frac{2x}{1+x^2}dx$$
$$= \left[\mathrm{Tan}^{-1}x + \frac{1}{2}\log(1+x^2)\right]_0^{\sqrt{3}}$$
$$= \frac{\pi}{3} + \frac{1}{2}\log 4 = \frac{\pi}{3} + \log 2$$